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Policy Equilibrium and Generalized Metarationalities for Multiple Decision-Maker Conflicts

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***Abstract*—A policy equilibrium is defined, and its properties investigated, for conflicts with more than two decision makers (DMs). A fundamental construction is the metarational tree, which expresses DMs' interactions as sequences of rounds, each consisting of an initial move by the focal DM followed by countermoves by the opponents. Using the metarational tree, the stability definitions of the graph model for conflict resolution can be adapted to apply to policies. These generalized metarational stabilities are shown to generalize Nash, general metarational, and symmetric metarational stabilities. Relationships among generalized metarationalities are derived, as are their connections with policy equilibria. Finally, the refinement that allows only credible moves (moves that are in the immediate interest of the mover) produces a new family of credible generalized metarational stabilities that generalizes the concept of sequential stability in the graph model.**

***Index Terms*—Conflict, graph model for conflict resolution, metarational tree, multiple decision makers (DMs), policy analysis, policy equilibrium.**

I. Introduction

In STRATEGIC conflicts, decision makers (DMs) often think in terms of how they will behave in situations that may arise in the future. Consider, for example, North Korea's continued development of nuclear weapons and missile capabilities, which threatens neighboring states, as well as the U.S. North Korea seems to have adopted the policy of maintaining its arms development programs as long as other countries do not help it meet its energy needs, and responding in force if attacked. Likewise, the other actors in this conflict may have formulated policies setting out how they would respond to any eventuality.

In this paper, such policies are studied formally for general cases involving two or more DMs, extending the concept of policy equilibrium developed earlier for cases with exactly two DMs [23], [24]. The context of this new approach to policy analysis is the graph model for conflict resolution [2], [3], [12], which uses a set of directed graphs to describe the decision options of conflict participants.

A policy is analogous to a strategy in game theory in that it specifies an action for each state in a conflict. With few exceptions, game theory approaches require that the DMs, called players, act either in a specific sequence (extensive-form game) or simultaneously (normal-form game). (Hamilton and Slutsky [8] suggest an approach that avoids this strong requirement in one restricted context.) In the real world, however, DMs may sometimes choose to act in any sequence, or simultaneously, or not at all. Conflicts in which timing and sequence are indefinite—many negotiations, for example—are not so easily modeled by games [1]. Moreover, game models require that players' preferences over outcomes be represented cardinally by von Neumann–Morgenstern utilities [22]. This requirement makes game models difficult to calibrate, as utilities are difficult to measure. Furthermore, there are instances when a DM's preferences (determined by voting, for example) are not transitive. In the preface of [19], Raiffa states that "I found the assumptions made in standard game theory too restrictive for it to have wide applicability."

The graph model for conflict resolution is designed to complement classical game theory in a way that avoids these and other problems. In a graph model, the conflict begins at a status quo state and progresses from state to state via state transitions controlled by various DMs, who may act whenever they choose to. Formally, a graph model represents the state transitions controlled by each DM as a directed graph with the set of states as the vertex set. The graph model incorporates many submodels of how a DM decides whether to move the conflict from its current state and, if so, to which state to move. These submodels, called stability definitions, allow for variation in many aspects of decision styles, such as level of foresight and level of risk aversion. Among these submodels, metarational stability (general and symmetric), sequential stability and limited-move stability have computational advantages, and are widely used to analyze real-world conflicts [4], [5], [9], [13], [18]. Recent extensions to the graph model methodology include robustness analysis [20] and techniques for handling preference uncertainty [15].

The goal of this paper is to design a procedure to identify states that are stable as a direct consequence of DMs' policies, within the framework of a multi-DM graph model. Specifically, the family of generalized metarational stabilities is defined using the metarational tree, a device that describes possible

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strategic interactions over a series of rounds. The properties of generalized metarationalities are compared in a fashion which is analogous to the study by Fang *et al.* [2] of graph model stability definitions. A DM's policy is called credible if it requires that moves be made to more preferred states only. Assuming credible policies, the authors define the credible policy equilibrium, credible metarational tree, and credible generalized metarational stabilities. Relationships between credible policy equilibria and credible generalized metarationalities are then derived. Finally, an existence theorem for policies with various forms of credible generalized metarationality demonstrates their relationship with the graph-model concept of sequential stability.

The remaining part of this paper is constructed as follows. Subsequent to the definition of a policy equilibrium (Section II-B), the metarational tree and generalized metarational stabilities are defined in Sections III-B and C, respectively. Relationships among stability definitions, generalized metarationalities, and policy equilibria are derived in Section IV and summarized in Fig. 3. The study of stability and equilibria under the restriction of credible moves is carried out in Section V.

II. Policy

The context for the policy definitions presented in Section II-B is the graph model for conflict resolution, which is outlined in Section II-A.

A. Graph Model for Conflict Resolution

A graph model for a strategic conflict consists of 1) a set of DMs, 2) a set of states, 3) for each DM, a directed graph with the set of states as vertices, and 4) for each DM, a preference structure on the set of states. Let $N = \{1, 2, \ldots, n\}$ represent the set of DMs and $S = \{s_1, s_2, \ldots, s_u\}$ the set of states. The finite directed graph $D_i = (S, A_i)$, $i \in N$, keeps track of the movements among states that DM i can make in one step. The vertices (nodes) of the graph represent the possible states (scenarios) of the conflict. If DM i can unilaterally move (in one step) from state s_1 to state s_2, there is an arc with orientation from s_1 to s_2 in A_i and DM i can reach state s_2 from state s_1. For $i \in N$, DM i's reachable list for state $s \in S$ is the set $R_i(s)$ of all states which DM i can reach from state s.

The preferences of DM i over the set of states is described by a pair of binary relations $\{\succ_i, \sim_i\}$ on S, where $s_1 \succ_i s_2$, for $s_1, s_2 \in S$, indicates that DM i prefers s_1 to s_2, and $s_1 \sim_i s_2$ means that DM i is indifferent between s_1 and s_2. It is assumed that $\succ_i$ is asymmetric, that $\sim_i$ is reflexive and symmetric, and that $\{\succ_i, \sim_i\}$ is strongly complete.

Sometimes, the notation $s_1 \succeq_i s_2$ is employed to indicate that either $s_1 \succ_i s_2$ or $s_1 \sim_i s_2$. Note that transitivity of preferences is not assumed, so that the key results in this paper are valid for both intransitive and transitive preferences.

A unilateral improvement from a specific state for a particular DM is any preferred state to which the DM can unilaterally move. The unilateral improvement list for DM i from state s is denoted as $R_i^+(s) = \{s_1 \in R_i(s) | s_1 \succ_i s\}$. Similarly, define $R_i^-(s) = \{s_1 \in R_i(s) | s \succ_i s_1\}$ and $R_i^=(s) = \{s_1 \in R_i(s) | s_1 \sim_i s\}$. Obviously, $R_i(s) = R_i^+(s) \cup R_i^-(s) \cup R_i^=(s)$.

B. Policy Sequences and Policy Equilibrium

A DM in a conflict may announce in advance what he or she intends to do at each state that could arise. For example, in a labor-management negotiation, the labor union may declare that it will go on strike if all of its demands are not met by the company. The company, in turn, may have a policy to lock out the union if it does not reduce its demands. Such a declaration, or policy, is clearly intended to influence the final result of the dispute.

Formally, a policy of DM i is a function $\mathcal{P}_i : S \to S$, such that $\mathcal{P}_i(s) \in R_i(s) \cup \{s\}$ and, therefore, $\mathcal{P}_i = \{\mathcal{P}_i(s), s = s_1, s_2, \ldots, s_u\}$ [23], [24]. Thus, a policy for a DM specifies what his or her action will be at each state (stay at that state or move to another state) if that state arises.

Now, we consider the possible interaction of the DMs in a sequence of moves and countermoves, in which no DM moves twice consecutively. For example, given an initial state s_0, a DM i may move from s_0 according to his or her policy $\mathcal{P}_i$. Then, another DM $j \in N - i$ may move from $\mathcal{P}_i(s_0)$ to another state $\mathcal{P}_j(\mathcal{P}_i(s_0))$. Depending on j's move, yet another DM $p \in N - j$, possibly i again, may move from $\mathcal{P}_j(\mathcal{P}_i(s_0))$ to $\mathcal{P}_p(\mathcal{P}_j(\mathcal{P}_i(s_0)))$, and so on.

A sequence of moves can be defined as a sequence $[s_0, i_1, s_1, i_2, \ldots]$ where $s_k \in S$ for $k = 0, 1, \ldots$ and $i_k \in N$ for $i = 1, 2, \ldots$, where i_k is called the moving DM of s_{k-1} for $k = 1, 2, \ldots$. In addition, a sequence is required to satisfy $i_{k+1} \neq i_k$ and $s_{k+1} \in \mathcal{P}_{i_{k+1}}(s_k)$. However, a DM might move several times if he or she does not move in succession. Therefore, in a 3-DM conflict, there might be a sequence in which DM 1 and DM 2 move several times before DM 3 makes any move.

A sequence may be finite or infinite. A finite sequence of length h is described as $[s_0, i_1, s_1, i_2, \ldots, i_h, s_h]$. A sequence terminates at s if the moving DM of s chooses to stay. The result of a sequence of length h or a terminated sequence is the last state. For an infinite sequence, there is at least one state that repeats infinitely, because the number of all states in a conflict model is finite. The result of an infinite sequence is defined to be the first infinitely repeating state. This definition can be justified by considering a move to have an infinitesimal cost as reflected in the inertia assumption defined by Kilgour and Zagare [14, p. 94] and the rational termination assumption of Brams [1, p. 27]. Notice that only ordinal preference information is used in the graph model and cardinal utilities are not assumed. Therefore, the average payoff of some or all infinitely repeating states cannot be calculated.

Given a state s_0 and a series of DMs $i_1, i_2, \ldots$, and the policies of DMs, a sequence of moves is obtained: $[s_0, i_1, s_1, i_2, \ldots]$, where $s_k = \mathcal{P}_{i_k}(s_{k-1})$. Note that given the DMs' policies, there may be more than one sequence, where each corresponds to a series of DMs.

The concepts of a policy equilibrium and policy stable state (PSS) for the case of two DMs are provided by Zeng *et al.* [23], [24]. These ideas can be generalized for the case of two or more DMs as follows.

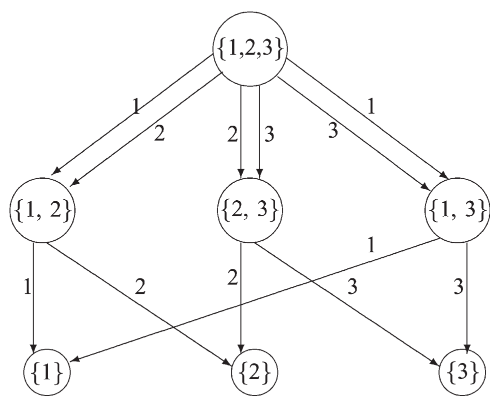

Fig. 1. Integrated graph for the truel game.

Definition 1: Policies $\mathcal{P}_1, \ldots, \mathcal{P}_n$ form a policy equilibrium with respect to the status quo state s^* if we have the following.

1) $\mathcal{P}_i(s^*) = s^*$ holds for all $i = 1, \ldots, n$.
2) For all i and $\mathcal{P}'_i$ such that $\mathcal{P}'_i(s^*) \neq s^*$, there is at least one sequence of moves with respect to policies $\mathcal{P}_1, \ldots, \mathcal{P}_{i-1}, \mathcal{P}'_i, \mathcal{P}_{i+1}, \ldots, \mathcal{P}_n$ and status quo state s^* such that the result of this sequence is not preferred to s^* by DM i.

A state s^* satisfying the above two conditions is called a PSS. The set of all PSSs is denoted by S^{PSS}.

Example 1: Consider the truel game [11] in which each of three DMs can either fire or not at either of the other DMs. The objectives of each DM are the following: 1) the DM, himself or herself, prefers to survive over being eliminated and 2) the DM would like to see fewer of his or her opponents remain alive. The first objective has a higher priority (i.e., is lexicographically more important than the second objective). It is assumed that each DM is a perfect shot, each shot eliminates one opponent, and no cooperation is allowed.

Since the three DMs can shoot or move in any order, it is not straightforward to describe this conflict using classical game theory. Furthermore, it is difficult to estimate cardinal preferences over the possible states. In contrast, the graph model and the developments in this paper can be easily applied to the truel. For convenience, three individual directed graphs D_i $(i = 1, 2, 3)$ are combined into an integrated graph as shown in Fig. 1, where a state is represented by the set of all surviving DMs and the number i beside an arc indicates that the arc is a member of A_i in graph D_i.

The DMs' preferences are as follows:

$$\begin{aligned}\{1\} \succ_1 \{1,2\} \sim_1 \{1,3\} &\succ_1 \{1,2,3\}\\ &\succ_1 \{2\} \sim_1 \{3\} \succ_1 \{2,3\}\\ \{2\} \succ_2 \{1,2\} \sim_2 \{2,3\} &\succ_2 \{1,2,3\}\\ &\succ_2 \{1\} \sim_2 \{3\} \succ_2 \{1,3\}\\ \{3\} \succ_3 \{1,3\} \sim_3 \{2,3\} &\succ_3 \{1,2,3\}\\ &\succ_3 \{1\} \sim_3 \{2\} \succ_3 \{1,2\}.\end{aligned}$$

Let $\{1,2,3\}$ be the status quo state. Consider the following policy $\mathcal{P}_i$ of DM $i = 1, 2, 3$:

$$\mathcal{P}_i(s) = \begin{cases} s - \{j\}, & \text{if } s = \{i, j\}, \text{ where } j \neq i\\ s, & \text{otherwise.}\end{cases}$$

In words, DM i fires at DM j if i and j are the only remaining DMs, and does not shoot at other states. These policies form a policy equilibrium and the status quo state $\{1, 2, 3\}$ is a PSS. In fact, if a DM (say DM 1) changes his or her policy and fires at another DM (say DM 2) at the states quo state, then DM 3 will shoot at DM 1 according to $\mathcal{P}_3$, which terminates the sequence of moves because no one can move again. The result of this sequence is state $\{3\}$, which is less preferred than the status quo by DM 1.

III. Generalized Metarationalities

In Section III-A, various types of unilateral moves and stability definitions are presented following a more detailed explanation given in [3]. The definition of the meterational tree in Section III-B forms the framework for defining general kinds of metarational stable states and resolutions in Section III-C.

A. Nash Stability and General and Symmetric Metarationalities

Let H be a subset of DMs and denote $R_H(s)$ [respectively $R^+_H(s)$] as the set of results of all possible sequences of moves (respectively unilateral improvements), by some or all of the DMs in H, starting from state s. Each member of $R_H(s)$ [respectively $R^+_H(s)$] is called a unilateral move (respectively unilateral improvement) by H. When H consists of only one DM i, then $R_H(s)$ [respectively $R^+_H(s)$] is also denoted by $R_i(s)$ [respectively $R^+_i(s)$], which is consistent with our notations in Section II-A.

The first stability definition is based on the idea of Nash [16], [17].

Definition 2: Let $i \in N$. A state $s^* \in S$ is Nash stable for DM i, denoted by $s^* \in S_i^{\mathrm{Nash}}$, iff $R^+_i(s^*) = \emptyset$. A state s^* is called a Nash resolution, denoted by $s^* \in S^{\mathrm{Nash}}$, iff it is Nash stable for all DMs.

The second and the third stability definitions are based on Howard's work [10].

Definition 3: For $i \in N$, a state $s^* \in S$ is general metarational (GMR) for DM i, denoted by $s^* \in S_i^{\mathrm{GMR}}$, iff for every $s_1 \in R^+_i(s^*)$ there is at least one state $s_x \in R_{N-i}(s_1)$ with $s_x \preceq_i s^*$. A state s^* is called a general metarational resolution, denoted by $s^* \in S^{\mathrm{GMR}}$, iff it is general metarational for all DMs.

Definition 4: Let $i \in N$. A state $s^* \in S$ is symmetric metarational (SMR) for DM i, denoted by $s^* \in S_i^{\mathrm{SMR}}$, iff for every $s_1 \in R^+_i(s^*)$, there exists $s_x \in R_{N-i}(s_1)$, such that $s_x \preceq_i s^*$ and $s_2 \preceq_i s^*$ for all $s_2 \in R_i(s_x)$. A state s^* is called a symmetric meterational resolution, denoted by $s^* \in S^{\mathrm{SMR}}$, iff it is SMR for all DMs.

B. Metarational Tree

Given policies $\mathcal{P}_j$ of all DM $j \neq i$, we consider the decision problem of DM i at state $s = s^1$ in an n-person conflict. If i seizes the initiative and moves, for example to state $s^1_1 \in R_i(s^1)$, then another DM $j^1 \in N - \{i\}$ moves from s^1_1 to

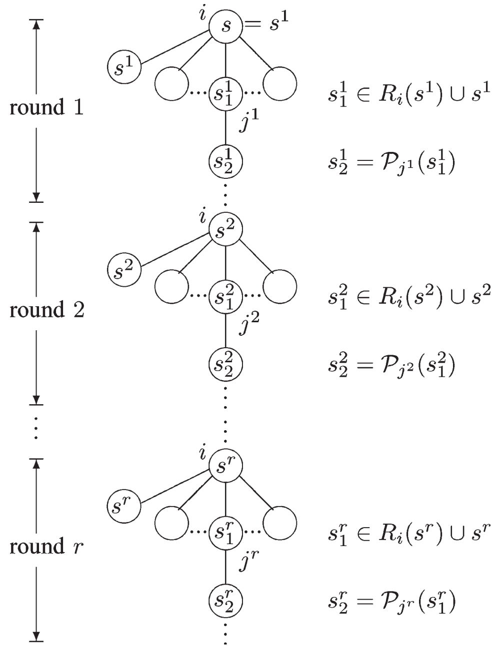

Fig. 2. DM i's metarational tree, given DM j's policy $\mathcal{P}_j$ for all $j \in N - i$.

$s_2^1 = \mathcal{P}_{j^1}(s_1^1) \in R_{j^1}(s_1^1)$. Depending on j^1's move, yet another DM $k^1 \in N - \{i, j^1\}$ might move from s_2^1 to $s_3^1 = \mathcal{P}_{k^1}(s_2^1) \in R_{k^1}(s_2^1)$, and so on. Note that, at state s_k^1 $(k = 2, 3, \ldots)$, DM i also has the opportunity to make another decision by staying at s_k^1 or moving to a state in his or her reachable list given by $R_i(s_k^1)$. If i moves from state s_k^1 (redenoted by s^2), for example to s_1^2, then other DMs may move from s_1^2 according to their policies, and so on. The above discussion is depicted in Fig. 2.

Given DM i, a sequence is said to be of r rounds with respect to DM i if there are r states in the sequence at which DM i moves. Notice that, whenever DM i moves, a new round is entered. Within each round, other DMs can move more than once as long as two moves by a DM are not in succession. Consider a sequence of r rounds with respect to DM i. We are interested in two kinds of sequences within r rounds. The first one, called i-sequence, ends with DM i as the last mover, and another, called $\bar{i}$-sequence, ends when other DMs do not move further (more precisely, $\bar{i}$-sequence ends just before entering the $r+1$ round). In the truel conflict in Example 1, only one round is allowed, since no DM can move twice consecutively. In an i-sequence, DM i shoots somebody and ends the sequence of moves. In contrast, in an $\bar{i}$-sequence, the other remaining DM is allowed to shoot DM i after DM i shoots a DM.

The two kinds of sequences inspire two kinds of stabilities. If all the resulting states of all i-sequences (respectively $\bar{i}$-sequences) in the metarational tree of r rounds are less than or equally preferred by DM i to the original state s, then DM i does not have the incentive to move away from s.

C. Metarationally Stable States of Round r

Definition 5: State s^* is called i-metarationally stable (respectively $\bar{i}$-metarationally stable) with r rounds for DM i, denoted by $s^* \in S_i^{\mathrm{MR}_r}$ (respectively $s^* \in S_i^{\overline{\mathrm{MR}}_r}$), if for each $s \in R_i(s^*)$, there is a set of policies $\mathcal{P}_j$ of all $j \in N - i$, and an i-sequence (respectively $\bar{i}$-sequence) starting with $[s^*, i, s]$ of r rounds or shorter such that the result of this sequence is not more preferred to s^* by DM i. A state s^* is called MR_r (respectively $\overline{\mathrm{MR}}_r$) resolution, denoted by $s^* \in S^{\mathrm{MR}_r}$ (respectively $s^* \in S^{\overline{\mathrm{MR}}_r}$), iff it is MR_r (respectively $\overline{\mathrm{MR}}_r$) stable for all DMs.

IV. RELATIONSHIPS AMONG STABILITY CONCEPTS

Using the structure of the metarational tree, two theorems are given for describing interesting relationships among stability definitions while a third theorem connects PSSs to certain stability definitions.

Theorem 1: 1) Nash stability is equivalent to MR_1 stability; 2) General metarationality is equivalent to $\overline{\mathrm{MR}}_1$ stability; 3) Symmetric metarationality is equivalent to MR_2 stability.

Proof: 1) Recall that in the MR_1 stability for DM i, only DM i is allowed to move once. It is evident that MR_1 stability is equivalent to the Nash stability for DM i.

2) Suppose that s^* is general metarational for DM i. If $R_i^+(s^*) = \emptyset$, then specify all policies of $j \neq i$ staying at all states. If i moves from s^* to $s_1 \in R_i(s^*)$, then any $\bar{i}$-sequence of round 1 has s_1 as its result, which is not more preferred to s^* by DM i. Therefore, s^* is $\overline{\mathrm{MR}}_1$ stable. If $R_i^+(s^*) \neq \emptyset$ and let $s_1 \in R_i^+(s^*)$, then there is a state $s_x \in R_{N-i}(s_1)$ such that $s_x \preceq_i s^*$. In other words, there is a sequence with moves by $N - i$ whose result s_x is not more preferred to s^* by DM i. We take this sequence as the shortest one (containing the least number of states). Then, each DM moves at most once at a state in this sequence. Note that DM i is not involved in this sanction sequence. This sequence can be used to specify a policy of DM j by

$$\mathcal{P}_j(s) = \begin{cases} s', & \text{if } [s, j, s'] \text{ is a part of the sequence} \\ s, & \text{otherwise.} \end{cases}$$

Then, the sanction sequence becomes an $\bar{i}$-sequence of round 1. In this way, s^* is $\overline{\mathrm{MR}}_1$ stable for DM i.

On the other hand, assume that s^* is $\overline{\mathrm{MR}}_1$ stable for DM i, then it is evidently general metarational for DM i, since a deviation of DM i will be followed by an $\bar{i}$-sequence as a sanction sequence.

3) Similar to the case of general metarationality. ■

Theorem 2: 1) If a state s is MR_r unstable for DM i, then it is also $\mathrm{MR}_1, \mathrm{MR}_2, \ldots,$ and MR_{r-1} unstable for DM i. Furthermore, if s is not an MR_r resolution, then it is also not an $\mathrm{MR}_1, \mathrm{MR}_2, \ldots,$ and MR_{r-1} resolution.

2) If a state s is $\overline{\mathrm{MR}}_r$ stable for DM i, then it is also $\overline{\mathrm{MR}}_1, \overline{\mathrm{MR}}_2, \ldots,$ and $\overline{\mathrm{MR}}_{r-1}$ stable for DM i. Furthermore, if s is an $\overline{\mathrm{MR}}_r$ resolution, then it is also an $\overline{\mathrm{MR}}_1, \overline{\mathrm{MR}}_2, \ldots,$ and $\overline{\mathrm{MR}}_{r-1}$ resolution.

Proof: 1) If state s^* is MR_r unstable for DM i, then for any given policies $\mathcal{P}_j$ of all other $j \neq i$, there is a deviation from s^* to s^0 of DM i, such that the result of any i-sequence of round r or shorter is more preferred to s^* by DM i. Given

$l < r$, if all those i-sequences are of round l or shorter, then s^* is MR_l unstable. Otherwise, let

$$\begin{bmatrix} s^*, i, s_1^{1,m}, i_2^{1,m}, s_2^{1,m}, \ldots, \\ \ddots \\ s^{l,m}, i, s_1^{l,m}, i_2^{l,m}, s_2^{l,m}, \ldots, \\ \ddots \\ s^{k,m}, i, s_1^{k,m}, i_2^{k,m}, s_2^{k,m}, \ldots \end{bmatrix}_{m=1,\ldots,M}$$

be all the i-sequences of round k with $l < k \leq r$, whose results are more preferred to s^* by DM i. Then, one claims that $s^{l,m} \succ_i s$ for at least one m. Otherwise, $s^{l,m} \preceq_i s$ for all $m = 1, \ldots, M$. Then, if all DMs $j \neq i$ change their policies to stay at $s^{l,m}$ for all m, the results of all the above i-sequences will not be more preferred to s^* by DM i, which contradicts to the MR_r stability for DM i. Based on the conclusion, it is evident that if s is not an MR_r resolution, then it is not an $\mathrm{MR}_1, \mathrm{MR}_2, \ldots,$ and MR_{r-1} resolution.

2) Let s^* be an $\overline{\mathrm{MR}}_l$ unstable for DM i for some $l \in \{1, \ldots, r-1\}$. Then, given any policies $\mathcal{P}_j$ of other players j, there is an $\bar{i}$-sequence of round l or a shorter terminated one, whose result $\bar{s}$ satisfies $\bar{s} \succ_i s^*$. If the sequence is terminated, then s becomes $\overline{\mathrm{MR}}_h$ unstable for DM i. Otherwise, i is the next mover since the sequence is an $\bar{i}$-sequence. Then, i can simply stay at $\bar{s}$, which terminates the sequence with result $\bar{s}$ better than s^* to i. Therefore, s^* is $\overline{\mathrm{MR}}_h$ unstable for DM i. Based on this conclusion, it is evident that if s is an $\overline{\mathrm{MR}}_r$ resolution, then it is also an $\overline{\mathrm{MR}}_1, \overline{\mathrm{MR}}_2, \ldots,$ and $\overline{\mathrm{MR}}_{r-1}$ resolution. ∎

Theorem 3: It holds that $S^{\mathrm{MR}_{r_1}} \subseteq S^{\mathrm{PSS}} \subseteq S^{\overline{\mathrm{MR}}_{r_2}}$ for all $r_1, r_2 = 1, 2, \ldots$

Proof: 1) This part shows $S^{\mathrm{MR}_{r_1}} \subseteq S^{\mathrm{PSS}}$. Contrary to the conclusion, let state s^* be an MR_r resolution, but not a PSS. By definition of PSS, there is a DM i, such that for an arbitrarily given policy $\mathcal{P}_j^\sharp$ of all DM $j \neq i$, there exists a policy $\mathcal{P}_i'$ of DM i which moves away from s^* and a sequence with respect to $\mathcal{P}_i'$ and $\{\mathcal{P}_j\}_{j \in N-i}$ whose result is $\bar{s}$ and

$$\bar{s} \succ_i s^*. \tag{1}$$

Specifically, let $\mathcal{P}_j^\sharp$ stay at all states s satisfying $s \preceq_i s^*$ for all DM j. Rename the sequence as

$$\begin{bmatrix} s^1 = s^*, i, s_1^1, i_2^1, s_2^1, \ldots, \\ s^2, i, s_1^2, i_2^2, s_2^2, \ldots, \\ \ddots \\ s^r, i, s_1^r, i_2^r, s_2^r, \ldots, \\ \ddots \end{bmatrix}.$$

It contains an i-sequence of round r as a subsequence, whose final state is s_1^r. If $s_1^r \preceq_i s^*$ for some r, then DM j stays at s_1^r according to his or her policy $\mathcal{P}_j^\sharp$. Hence, the sequence becomes terminated and the result is $s_1^r \preceq_i s^*$, which contradicts (1). Therefore, $s_1^r \succ_i s^*$ for all r and one concludes that state s^* is MR_r unstable for i.

2) This part shows $S^{\mathrm{PSS}} \subseteq S^{\overline{\mathrm{MR}}_{r_2}}$. Contrary to the conclusion, suppose that a PSS s^* with policies $\mathcal{P}_1^*, \ldots, \mathcal{P}_n^*$ is not an $\overline{\mathrm{MR}}_r$ resolution, for example $\overline{\mathrm{MR}}_r$ unstable for DM i, for a positive integer r. Then, with respect to $\mathcal{P}_j^*$, there is a policy $\mathcal{P}_i^\sharp$ and an $\bar{i}$-sequence of round r, or a shorter terminated one with a result more preferred to s^* by DM i.

1) If this sequence is a terminated one, then the result of this sequence is also the result of policies $\mathcal{P}_i^\sharp$ and $\mathcal{P}_j^*$ (for all $j \in N - i$) with respect to status quo s^* and first mover i, which contradicts the fact that s^* is a PSS with policies $\mathcal{P}_1^*, \ldots, \mathcal{P}_n^*$.
2) If this sequence is not a terminated one, revise $\mathcal{P}_i^\sharp$ a little to stay at all states more preferred to s^*. By adding i at the end of the sequence, a shorter terminated sequence is obtained. Then, the above arguments derive a contradiction again. ∎

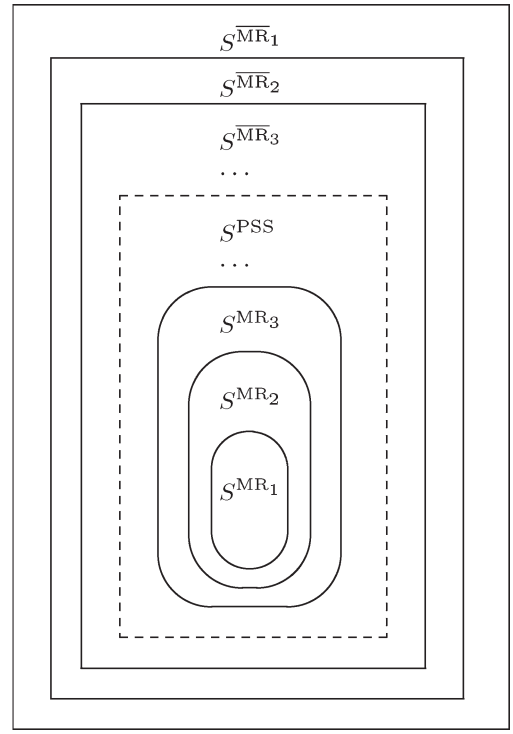

Fig. 3. Relationships among S^{MR_r}, $S^{\overline{\mathrm{MR}}_r}$ and S^{PSS}.

The relationships established in Theorems 2 and 3 are displayed in Fig. 3. In Example 1, $\{1, 2, 3\}$ is a member of S^{PSS} and $S^{\overline{\mathrm{MR}}_1}$ but is not a member of S^{MR_1}.

V. Credible Policy and Credible Metarationalities

A policy of a DM may contain some moves going to less preferred states. A DM's policy is deemed to be credible, if he or she always moves to a more preferred state. Incredible moves are excluded in the refinement of Nash equilibria in game theory, such as the subgame perfect equilibrium of Selten [21]. Hence, a credible policy is defined as $\mathcal{P}_i^c(s) \in R_i^+(s) \cup \{s\}$. The policy $\mathcal{P}_i$ in Example 1 is credible since each DM either stays at the state s or only moves to a more preferred state. By requiring a policy to be credible, one obtains a credible MR_r (respectively $\overline{\mathrm{MR}}_r$) denoted by CMR_r (respectively $\overline{\mathrm{CMR}}_r$). The credible metarational tree shown in Fig. 4 is constructed in a fashion that is similar to the metarational tree

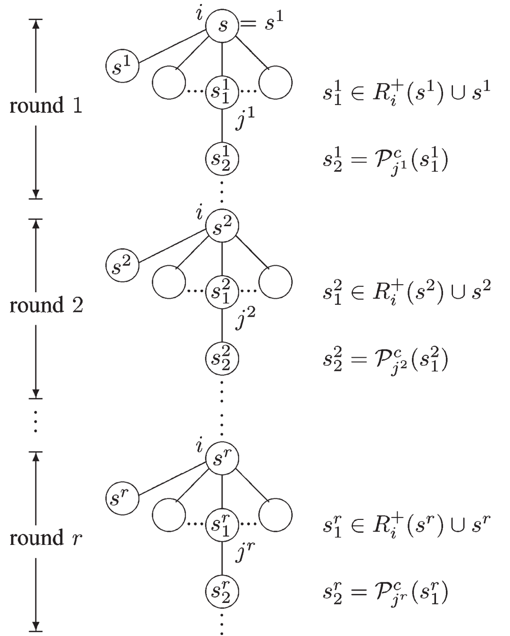

Fig. 4. DM i's credible metarational tree given $\mathcal{P}_j^c$ of all $j \in N - i$.

in Fig. 2. Notice that all the moves in Fig. 4 are restricted to credible ones.

Definition 6: State s^* is called i-credibly metarationally stable (respectively $\bar{i}$-credibly metarationally stable) with r rounds for DM i, denoted by $s^* \in S_i^{\mathrm{CMR}_r}$ (respectively $s^* \in S_i^{\overline{\mathrm{CMR}}_r}$), if for each $s \in R_i^+(s^*)$, there is a set of credible policies $\mathcal{P}_j^c$ of all $j \neq i$, and an i-sequence (respectively $\bar{i}$-sequence) starting with $[s^*, i, s]$ of r rounds or shorter such that the result of this sequence is not more preferred to s^* by DM i. A state s^* is called CMR_r (respectively $\overline{\mathrm{CMR}}_r$) resolution, denoted by $s^* \in S^{\mathrm{CMR}_r}$ (respectively $s^* \in S^{\overline{\mathrm{CMR}}_r}$), iff it is CMR_r (respectively $\overline{\mathrm{CMR}}_r$) stable for all DMs.

Similar to the conclusions of Theorem 1, the concept of CMR_1 stability is equivalent to the MR_1 stability, or the Nash stability. The concept of $\overline{\mathrm{CMR}}_1$ stability (see round 1 of Fig. 4) is equivalent to the following sequential stability of Fraser and Hipel [6], [7].

Definition 7: For $i \in N$, a state $s^* \in S$ is sequentially stable (SEQ) for DM i, denoted by $s^* \in S_i^{\mathrm{SEQ}}$, iff for every $s_1 \in R_i^+(s^*)$ there is at least one state $s_x \in R_{N-i}^+(s_1)$ such that $s_x \preceq_i s^*$. A state s^* is called a sequential resolution, denoted by $s^* \in S^{\mathrm{SEQ}}$, iff it is sequentially stable for all DMs.

Similar to Theorem 2, the following theorem can be derived.

Theorem 4: 1) If a state s is CMR_r unstable for DM i, then it is also $\mathrm{CMR}_1, \mathrm{CMR}_2, \ldots,$ and CMR_{r-1} unstable for DM i. Furthermore, if a state s is not a CMR_r resolution, then it is also not a $\mathrm{CMR}_1, \mathrm{CMR}_2, \ldots,$ and CMR_{r-1} resolution. 2) If a state s is $\overline{\mathrm{CMR}}_r$ stable for DM i, then it is also $\overline{\mathrm{CMR}}_1, \overline{\mathrm{CMR}}_2, \ldots,$ and $\overline{\mathrm{CMR}}_{r-1}$ stable for DM i. Furthermore, if a state s is a $\overline{\mathrm{CMR}}_r$ resolution, then it is also a $\overline{\mathrm{CMR}}_1, \overline{\mathrm{CMR}}_2, \ldots,$ and $\overline{\mathrm{CMR}}_{r-1}$ resolution.

The following fact is evident.

Fact 1: If transitivity of movement holds for each DM i, then for any $H \subseteq N$ and state $s' \in R_H^+(s)$, it holds that $R_H^+(s') \subseteq R_H^+(s)$.

This fact is used in proving the following theorem.

Theorem 5: Given any positive integer r, if transitivity of movement and preferences holds, then $S^{\overline{\mathrm{CMR}}_r} \neq \emptyset$ for a multiple decision-maker conflict.

Proof: This proof is inspired by and generalizes the proof of [7, Th. 13.23]. If the theorem is false for a given r, each state is not $\overline{\mathrm{CMR}}_r$ for at least one DM (say i_1). Specifically

$$\text{let } s_0 \text{ be the state that is most preferred by DM } i_1 \text{ among all those that are } \overline{\mathrm{CMR}}_r \text{ unstable states for DM } i_1. \quad (2)$$

(Such a state s_0 exists because of transitivity of preferences.) Then, there is at least one $\bar{i}_1$-sequence of round r or a shorter terminated one, beginning from a move by DM i_1. Without loss of generality, assume the sequence is not terminated. Otherwise, the following argument holds for a smaller r. Let

$$\begin{bmatrix} s^{1,m} = s_0, i_1, s_1^{1,m}, i_2^{1,m}, s_2^{1,m}, \ldots, \\ s^{2,m}, i_1, s_1^{2,m}, i_2^{2,m}, s_2^{2,m}, \ldots, \\ \ddots \\ s^{r,m}, i_1, s_1^{r,m}, i_2^{r,m}, s_2^{r,m}, \ldots, \end{bmatrix}_{m=1,\ldots,M}$$

be all such kinds of sequences, and hence $s^{r,m} \succ_{i_1} s_0$ holds for all $m = 1, 2, \ldots, M$. One can claim that

$$s_1^{r,m} \succ_{i_1} s_0 \quad \text{for some} \quad m = 1, 2, \ldots, M. \quad (3)$$

Otherwise, all DMs $i_2^{r,m}$ for $m = 1, \ldots, M$ can use a policy which stays at all $s_1^{r,m}$ $(m = 1, 2, \ldots, M)$, so that there is no $\bar{i}_1$-sequence of round r whose result is more preferred to s_0 by DM i_1 and s_0 becomes $\overline{\mathrm{CMR}}_r$.

According to (2), $s_1^{r,m}$ is $\overline{\mathrm{CMR}}_r$ stable for DM i_1 for some $m = 1, 2, \ldots, M$ by (3). If $s_1^{r,m}$ is also $\overline{\mathrm{CMR}}_r$ stable for all $j \in N - i$, then the proof is completed. Otherwise, there is a DM (say i_2) and m such that $s_1^{r,m}$ is $\overline{\mathrm{CMR}}_r$ stable for i_1 but not $\overline{\mathrm{CMR}}_r$ stable for i_2. It is now shown that if $s \in R_{N-i_1}^+(s_1^{r,m}) \cup \{s_1^{r,m}\}$, then s is $\overline{\mathrm{CMR}}_r$ stable for DM i_1. In fact, if $s = s_1^{r,m}$, this has already been shown; otherwise, note first that $s \succ_i s_0$ (if this inequality were false, then a sanction sequence against i_1 departing s_0 is obtained), and the conclusion is true by (2).

Now, impose the induction assumption that, for $1 \leq k < n$, there are distinct DMs $i_1, i_2, \ldots, i_{k+1}$ and a state p^k such that p^k is not $\overline{\mathrm{CMR}}_r$ for DM i_{k+1} and if $s \in R_{N-i_1-\cdots-i_k}^+(p^k) \cup \{p^k\}$, then s is $\overline{\mathrm{CMR}}_r$ for $i_1, \ldots, i_k$. Let q^{k+1} denote a state of $R_{N-i_1-\cdots-i_k}^+(p^k) \cup \{p^k\}$ which is most preferred by i_{k+1} among all those that are not $\overline{\mathrm{CMR}}_r$ for i_{k+1}. There is at least one such state in $R_{N-i_1-\cdots-i_k}^+(p^k)$, namely p^k. By the definition of $\overline{\mathrm{CMR}}_r$, DM i_{k+1} has a deviation from q^{k+1} to a better state p^{k+1}, which cannot be sanctioned by an $\bar{i}_{k+1}$ sequence of round r. Observe by Fact 1 that $p^{k+1} \in R_{N-i_1-\cdots-i_k}^+(p^k)$ because $q^{k+1} \in R_{N-i_1-\cdots-i_k}^+(p^k)$ and $p^{k+1} \in R_{i_{k+1}}^+(q^{k+1})$. Since $p^{k+1} \succ_{i_{k+1}} q^{k+1}$, then p^{k+1} is $\overline{\mathrm{CMR}}_r$ for i_{k+1} by the definition of q^{k+1}. Furthermore, it is $\overline{\mathrm{CMR}}_r$ for $i_1, \ldots, i_k$ by the induction hypothesis. If either $k = n - 1$ or p^{k+1} is $\overline{\mathrm{CMR}}_r$ for all $N - i_1 - \cdots - i_k - i_{k+1}$, the proof is finished. Otherwise, there is a DM $i_{k+2} \in N - i_1 - \cdots - i_{k+1}$ such that p^{i+1} is not $\overline{\mathrm{CMR}}_r$ for i_{k+2}. To complete the induction, it is necessary

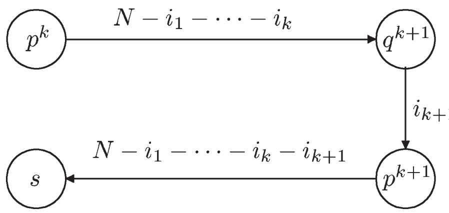

Fig. 5. States used in the proof of Theorem 5.

to show that if $s \in R^+_{N-i_1-\cdots-i_{k+1}}(p^{k+1}) \cup \{p^{k+1}\}$, then s is $\overline{\mathrm{CMR}}_r$ for all $i_1, \ldots, i_k, i_{k+1}$ (see Fig. 5 for an illustration of the notation). This assertion is true when $s = p^{k+1}$, as has already been shown. For other s, it holds that

$$\begin{aligned} s \in R^+_{N-i_1-\cdots-i_{k+1}}(p^{k+1}) &\subseteq R^+_{N-i_1-\cdots-i_k}(p^{k+1}) \\ &\subseteq R^+_{N-i_1-\cdots-i_k}(p^k) \end{aligned}$$

where the last relationship is from Fact 1. Therefore, s is $\overline{\mathrm{CMR}}_r$ for $i_1, \ldots, i_k$ by the induction hypothesis. Furthermore, since

$$s \in R^+_{N-i_1-\cdots-i_k-i_{k+1}}(p^{k+1}) \subseteq R^+_{N-i_{k+1}}(p^{k+1})$$

where the last relationship is from Fact 1, it holds that $s \succ_{i_{k+1}} q^{k+1}$, otherwise, s is the result of a credible sanction $\bar{i}_{k+1}$-sequence against i_{k+1}'s deviation from q^{k+1} to p^{k+1}. Therefore, s is $\overline{\mathrm{CMR}}_r$ for i_{k+1} by the definition of q^{k+1}. ■

VI. Conclusion

Policy equilibria and associated PSSs are clearly defined along with a new family of stability definitions for a conflict having multiple DMs. The metarational tree provides a general framework within which the family of stability concepts can be conveniently defined and mathematical relationships rigorously developed among stability definitions and PSSs. The first four theorems provide an understanding of interesting relationships among these new stability definitions while the last theorem guarantees the existence of at least one equilibrium for specific stability definitions. Because of the foregoing contributions, the methodology of the graph model for conflict resolution can now address a richer range of conflict situations. In practice, this implies that enhanced insights and better decision advice can be obtained using formal analyses for a particular conflict.

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